Are quantum 'irreality' and 'nonlocality' ineluctable?

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The early history of the development of Quantum Mechanics is surveyed to discern the arguments leading to the introduction of the notions of 'irreal' wave functions and 'nonlocal' correlations. It is argued that the assumption that Quantum Mechanics is 'complete', i.e., not just a variant of Statistical Mechanics, is the feature compelling the introduction of these otherwise problematic properties. Additionally, a consequence of the error first found by JAYNES in proofs of BELL's "theorem", is illustrated. Finally, speculation on the practical consequences of recognising that "entanglement" is a feature of all hyperbolic differential equations is proposed.
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I. HISTORY

Twentieth century physicists faced extraordinary challenges in terms of the scale of the phenomena to be explained. The extremely small sizes of the objects covered by Quantum Mechanics (QM) and the very large scale of events covered by Relativity, posed situations that were unimaginable within the then customary understanding and concepts of science. This fostered, out of desperation, license to introduce theretofore unacceptably exotic hypotheses, (e.g., quantisation, frame independent light velocity) for which there was only indirect laboratory evidence. At the same time, at least one desiderium was assumed, largely without deep reflection, namely, that the new theories under development were fundamental and complete (that is, that at their level they are theories of individual entities, not theories quantifying statistics of ensembles of such entities).

Herein, I shall argue that the last mentioned assumption, *completeness*, is the key underlying cause that the exotic (and arguably antirational) notions: *irreality* and *nonlocality,* have been ensconced in QM. The reasoning that led to this situation, was not concerned in the first instance with the philosophically problematic nature of these features, indeed the terms themselves, as well as their accepted technical denotation, appeared in the literature up to years later than the introduction of the mathematical structure to which they refer.

For the sake of expository efficiency, herein a reconstructed line of reasoning will be described that, I believe, is a composite of what happened, but not in the mind of any one person. The actual development, to the extent that the historically true story can be discerned at all in retrospect, was focused on finding the mathematical structure that mimicked those aspects of the studied phenomena accessible to experiment, which were nearly always imagined to be just a portion of what happens 'down deep'. In seeking these models or paradigms, however, it is clear that the founding fathers individually quite early settled on a personally preferred paradigm, and tilted their analysis to support it. For QM, the accumulated effect, warped by sociological factors, is what has become known as the "Copenhagen interpretation".

Some may be inclined to write off these concerns as 'mere philosophy' of little interest "during working hours" to physical scientists. However, one can observe that, wherever there is a 'philosophical' issue, the mathematics in use is also afflicted with one or another pathology. Such a coincidence makes perfect sense actually; mathematical problems (inconsistent or incomplete calculations) nearly always parallel verbal problems (erroneous syntax) insofar as mathematics is just the use of symbols and formalised algorithms as a sort of shorthand for ideas originally expressed verbally. Indeed, all mathematics is taught by means of oral explanations. Philosophical problems, following this logic, are often, therefore, symptoms of physics problems. Moreover, there may well be very practical consequences with regard to eventual applications derived from self consistent interpretations.

II. IRREALITY

Irreal wave functions are of the form:

$$\psi(r_1, r_2) = \varphi(r_1, r_2) + \chi(r_1, r_2), \qquad (1)$$

where $\psi(r_1, r_2)$ represents a wave function for a combined system of subsystems, and both $\varphi(r_1, r_2)$ and $\chi(r_1, r_2)$ are the wave functions for potential outcomes. Wave functions of this form are *irreal* if the summands are logically mutually exclusive, i.e., states that by all logic cannot exist simultaneously. There is, in addition, a continuous variant of this same structure, in which, for example a point particle, which can be at only one location at once, is represented by a wave packet finite over several locations. All the conceptual features of irreality are evident, nevertheless, in the binary version as captured in Eq. (1); thus, let us focus on it.

A prototypical example of a binary irreal wave function is the singlet state used to describe the emission of correlated photon pairs for an EPR experiment:

$$\psi(1, 2) = \frac{1}{\sqrt{2}} \left(\psi_1 S(\uparrow) \psi_2 S(\rightarrow) - \psi_1 S(\rightarrow) \psi_2 S(\uparrow) \right), \quad (2)$$

where the system's state is supposed to be the difference of permutations of polarised photon pairs. Since each pair can have one *or* the other orientation-combination at a time, the

summands are logically mutually exclusive. Nevertheless, according to the 'Copenhagen' interpretation, this combination state is considered the 'real' ontological state of the system until measurement 'collapses' this wave function to one or the other 'non irreal' summand—as must happen since irreal states are *never* actually observed in experiments.

Irreality of wave functions is nowadays of relatively low concern; an explanation of a possible reason for this should emerge below. In part, this is due to the fact that in many cases the summands are not mutually exclusive and the wave function exhibits simple, and non problematic, 'superposition.'

A natural question here is: just how did this situation arise; what reasoning lead to accepting such an extraordinary supposition? What problems brought this reasoning about? To this writer it appears that the answer should be found exactly there, where the first appearance of wave functions of the form of Eq. (1) arose in the literature. Almost certainly, it is in HEISENBERG's initial treatment of the two electron atom, helium.[1]

His initial efforts to solve this problem were aimed primarily at getting a useful answer for spectroscopy and only incidentally at developing and promulgating his preferred paradigm. As a 'test bed' for developing the appropriate formalism, HEISENBERG chose the problem of coupled harmonic oscillators.[1] This problem is parallel to the problem of the helium atom in that each electron is primarily influenced by the nucleus and only secondarily by the other electron, analogously to oscillators whose behaviour is primarily determined by the 'spring constant(s)' and secondarily by a relatively weak coupling between the oscillators.

HEISENBERG observed, that it is a characteristic feature of atomic systems, that the components of which they are comprised, namely electrons, are identical and subject to identical forces. Therefore, in order to invest this feature in his 'test bed', he assumed the HAMILTONian to be of the form:

$$H = \frac{1}{2m}p_1^2 + \frac{m}{2}\omega^2 q_1^2 + \frac{1}{2m}p_2^2 + \frac{m}{2}\omega^2 q_2^2 + m\kappa q_1 q_2; \quad (3)$$

i.e., the frequencies and masses of the coupled oscillators are taken to be identical. In Eq. (3), q_1, q_2 denote the coordinates, p_1, p_2 the momenta, m and ω the mass and frequency respectively, and κ the interaction constant. With help of the well known transformations:

$$q_1' = \frac{1}{\sqrt{2}}(q_1 + q_2), \quad q_2' = \frac{1}{\sqrt{2}}(q_1 - q_2), \quad (4)$$

Eq. (3) is transformed into the separated form:

$$H = \frac{1}{2m}p_1'^2 + \frac{m}{2}\omega_1'^2 + \frac{1}{2m}p_2'^2 + \frac{m}{2}\omega_2'^2 q_2'^2, \quad (5)$$

where

$$\omega_1'^2 = \omega^2 + \kappa, \quad \omega_2'^2 = \omega^2 - \kappa. \quad (6)$$

In other words, H separates into the sum of two oscillators, such that each corresponds to a "normal mode", in the technique long before developed by DANIEL BERNOULLI. When only the first mode, q_1', is excited, then both masses oscillate in phase, and when only q_2' is excited, out of phase.

The energies according to QM for the combined system are then give by the equation:

$$H_{n_1', n_2'} = \frac{\omega_1' h}{2\pi}\left(n_1' + \frac{1}{2}\right) + \frac{w_2' h}{2\pi}\left(n_2' + \frac{1}{2}\right), \quad (7)$$

where n_1' and n_2' are integers.

In his scheme, the solutions that HEISENBERG obtained are matrix elements found using his version of QM. The solutions from Eq. (7) are, as is usually the case for normal coordinates, not physically observable, but particular solutions of an abstract combined system. The observables are the inverses of Eqs. (4). At this stage the solutions do not yet suffer irreality; indeed, the classical mechanical solutions present no philosophical problems. If the initial conditions are appropriate, the system executes motion described by one of the normal modes, otherwise, the solution is a secular oscillation of the total system energy between the two oscillators.

Observing that, at the atomic scale it is not possible to determine the exact details of light absorption and emission, HEISENBERG asserted, not altogether cogently, that he considered discontinuities more faithful to reality than SCHRÖDINGER's *continuous* waves.[2] It is reasonably arguable, however, that actually he succumbed to sociological pressure, as portrayed by FORMAN, namely to conform to the pervasive antideterministic philosophical proclivities prevailing in German academia following World War I.[3] Thus, with scant underpinning, seemingly in order to accommodate the *Zeitgeist*, he simply *chose* a paradigm involving intrinsic randomness. This, HEISENBERG realized by supposing that the solutions, in place of secular oscillation, exhibit random, spontaneous, secular-like jumping back and forth.

Instantaneous jumping by itself, is not necessarily irreal; implicitly there can be a hidden variable that specifies as a function of time just which electron is excited in the series of jumps back and forth, that perhaps an extention of QM could predict. However, admitting this possibility would undermine the sociological goal of discrediting determinism; and so, for whatever reason, this possibility was rejected out of hand.

The explicit insinuation of the 'completion' assumption into the paradigm, or the 'Copenhagen' interpretation, was a complicated and turbid development, the history of which has been analysed extensively by BELLER.[4] Notably, VON NEUMANN took up the question of completion at the latest by 1932. He proffered a demonstration to the effect that presuming the existence of hidden variables completing QM implied that some existing quantum structure is objectively false.[5] Although this seemed to settle the question, it was quickly seen (but not widely heeded) that his argument contained irrelevant hypothetical inputs.[6] In general, both those supporting HEISENBERG's discontinuous and SCHRÖDINGER's continuous paradigms seemed more eager than not to assume that quantum theory *is* complete. Presumably, this happened, to some degree uncritically, as it satisfied the ambition of the participants to be creating a deep and fundamental new theory; and moreover, it did not clash with the prevailing cultural

bias.

The strictly logical consequence, the implicit paradox, of this assumption, however, was not assiduously analysed until later after the renowned paper by EINSTEIN, PODOLSKY and ROSEN (EPR).[7] It was only with the controversy evoked by their arguments that the consequences of 'completeness', became a generally acknowledged issue. For example, SCHRÖDINGER reacted immediately with analysis of the then current understanding of the meaning of QM in which he introduced the term "entanglement" for that form of correlation attributed to irreal wave functions.[8] In his paper on this matter there are no new quantum techniques introduced, just new terminology to facilitate deliberate analysis of the then just implicit connotations for the terms used discussing interpretations. This work, being overtly critical, was no doubt a contribution to the duel with HEISENBERG on the relative merits of discontinuous (matrix) versus continuous wave paradigms. In it SCHRÖDINGER embellished EPR's illustrative gedanken experiment to the now renowned and absurdly irreal live-dead "cat paradox".

The crucial point here is, if QM is complete, then there can be no hidden variables to specify which excited state among the constituents at any moment is ontologically valid, thereby giving their sum this role. That is, then all components, even mutually exclusive options, are to be extant simultaneously, even when not verifiable by observation. In short, if QM *is* complete, then there must be *irreal* states!

III. NONLOCALITY

Nonlocality was introduced as the cure for irreality. The fact that observations *never* (could!) reveal states that are comprised of irreal sums of mutually exclusive options, implies, it was hypothesised, that measurement itself somehow "collapses" the ontological wave function to the 'post-measurement wave function', that is, just one of the options comprising the irreal, 'pre-measurement' wave function.[1] Insofar as measurement of one of a correlated pair instantaneously collapses a wave function for the other, regardless of its separation, the process insinuates 'nonlocality'.

JOHN BELL in the 1950's, having rediscovered VON NEUMANN's misstep and with inspiration from BOHMian mechanics, took up the issue with the goal of bringing it to an experimental nexus.[9] He did this with analysis subsequently, and strictly incorrectly, labelled a "theorem", to the effect that locality demands that a certain statistic (Eq. (17) below) be less than $|2|$. Experiments show, however, that it can reach $|2\sqrt{2}|$; and, nowadays the difference is taken to characterise "stronger than classical" correlations which have become denoted "entanglements". In recent times this matter has taken on, so to

speak, a life of its own, that is, irreality has slid into oblivion and usually not discussed as the *raison d'être* for nonlocality

In any case, BELL's final conclusion was that, because QM is ineluctably nonlocal, any insinuation of hidden variables to 'complete' it, cannot lead to a deeper formulation that is 'local' and 'real'. In turn, however, BELL's argument too has come under criticism, starting with EDWIN JAYNES,[10], who parsed BELL's encoding of locality and found that it overlooked structure requiring BAYES' formula for *conditional* probabilities. This writer has taken up this line and extended it by working out explicit consequences of JAYNES' point for the experiments thought to verify BELL's analysis.[11; 12] That is, classical, local, realist models for all the generic forms of EPR-type experiments have been developed which lead to calculations, based essentially on MALUS' Law, utterly devoid of irreality and nonlocality, yielding curves precisely mimicking data taken in EPR experiments, $|2\sqrt{2}|$ and all. Since BELL's theorem states in effect that such models do not exist, exhibiting them shows that BELL's theorem is wrong or misunderstood.

JAYNES' essential point is that whereas BELL wrote the joint probability $P(a,b)$ for a coincidence event in an EPR experiment, where the measurement settings are a and b, and λ represents the imputed "hidden variables", in the form:

$$P(a,b) = \int d\lambda \rho(\lambda) P(a,\lambda) P(b,\lambda), \qquad (8)$$

he should have written:

$$p(a,b) = \int d\lambda \rho(\lambda) P(a|b,\lambda) P(b|\lambda), \qquad (9)$$

where the latter form employs what is known as BAYES' formula or simply as the definition of *conditional* (versus absolute) probability.[14] Eq. (9), it can be easily verified, does not admit deriving any form of the renowned "BELL inequalities". What this means is that BELL misencoded 'locality' as 'statistical independence', so that the observed violation of such inequalities in experiments cannot be interpreted to mean that nonlocal interaction or nonlocal correlation is in evidence. Rather, only, that the inequalities pertain when there is no correlations of any type, nonlocal or otherwise, contrary to EPR's, and subsequently to BELL's, hypothesis and the explicit design of experiments involving *correlated* pairs of inputs.

Since this matter has been explicated in detail elsewhere, [11; 12; 13], here only one variant of several counter arguments shall be featured. Its key idea is that if the physical meaning of the terms in BELL's extraction of his inequalities are carefully interpreted, it is seen that certain of them must be zero, thereby leading to a form of these inequalities for which there is no significance with regard to his sought after conclusion. This counter argument, which is independent of JAYNES' criticism, but based on the same structure, proceeds as follows:

First, recall a mathematical technicality concerning the product of two Dirac delta functions, which is essential for what follows. It is that the integral of the product of two delta functions for which the arguments are different, equals zero;

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[1] Although many early papers read as if their authors imagined wave 'collapse' upon measurement, VON NEUMANN appears to have been the first to have made the matter explicit.[5] The issue is confounded by the fact, that measurement is to collapse simple superpositions also.

i.e.:

$$\int dx\, f(x)\delta(x-l)\delta(x-m) = 0, \qquad (10)$$

whenever $l \neq m$.

The derivation of a Bell Inequality starts from BELL's fundamental assertion:

$$P(a,b) = \int d\lambda\, \rho(\lambda)A(a,\lambda)B(b,\lambda), \qquad (11)$$

where, per *explicit assumption:* A is not a function of b; nor B of a; and each represents the appearance of a photoelectron in its wing of an EPR experiment, and a and b are the corresponding polariser filter settings.[2] This is motivated on the grounds that a measurement at station A, if it respects 'local-ity', *so argues* BELL, cannot depend instantaneously on remote conditions, such as the settings of the other polariser. In addition, each, by definition, satisfies

$$|A| \leq 1, \;\; |B| \leq 1, \qquad (12)$$

which in this case effectively restricts the analysis to the case of just one photoelectron per time window per detector. Eq. (11) encodes the condition, that when the hidden variables are averaged out, the usual results from QM are to be recovered.

The λ above in BELL's analysis stands for a hypothetical set of "hidden variables", which, if they exist, should render QM deterministic. This set may include many different types of variables, such as discrete, continuous, tensor or whatever.

Extraction of inequalities proceeds by considering differences of two such correlations where (a, b), *i.e.*, the polariser axis of measuring stations, left and right, differ:

$$P(a,b) - P(a,b') = \int d\lambda\, \rho(\lambda)[A(a,\lambda)B(b,\lambda) - A(a,\lambda)B(b',\lambda)], \qquad (13)$$

to which zero in the form:

$$A(a,\lambda)B(b,\lambda)A(a',\lambda)B(b',\lambda) - A(a,\lambda)B(b',\lambda)A(a',\lambda)B(b,\lambda) = 0, \qquad (14)$$

is added to get:

$$P(a,b) - P(a,b') = \int d\lambda\, \rho(\lambda)A(a,\lambda)B(b,\lambda)[1 \pm A(a',\lambda)B(b',\lambda)] - \int d\lambda\, \rho(\lambda)A(a,\lambda)B(b',\lambda)[1 \pm A(a',\lambda)B(b,\lambda)], \qquad (15)$$

which, in turn, upon taking absolute values and in view of Eqs. (12), BELL wrote as:

$$|P(a,b) - P(a,b')| \leq \int d\lambda\, \rho(\lambda)[1 \pm A(a',\lambda)B(b',\lambda)] + \int d\lambda\, \rho(\lambda)[1 \pm A(a',\lambda)B(b,\lambda)]. \qquad (16)$$

Then, using Eq. (11), and the normalisation condition $\int d\lambda\, \rho(\lambda) = 1$, he got, for example:

$$|P(a,b) - P(a,b')| + |P(a',b') + P(a',b)| \leq 2, \qquad (17)$$

a 'Bell inequality'.

Now, however, if the λ are a complete set [2,3], thereby rendering everything deterministic so that all probabilities as functions of λ become Dirac or Kronecker delta distributions, then the A's and B's in Eq. (15) are pair-wise; that is to say as individual events comprising the generation at the source of one pair, are non zero for distinct values of λ, which, by virtue of completeness, do not coincide for distinct events, *i.e.*, for different pairs. That is, for each pair of settings $(a = r, b = s)$ and iteration of the experiment, n, there exists a unique set of values, $\lambda_{a=r,b=s}(n = \text{integer})$, or in more compact notation, $\lambda(n)$, say, for which $A(a|\lambda(n))B(b|\lambda(n))$ is non-zero (1 in the discrete case, ∞ in the continuous case). In other words, each product $A(a|\lambda(n))B(b|\lambda(n))$ can be written in the form $f(x)\delta(x - \lambda(n))$, so that all quadruple products, e.g.,

$$A(a|\lambda(n))B(b|\lambda(n))A(a|\lambda(m))B(b|\lambda(m)), \qquad (18)$$

are equivalent to the form:

$$f(x)\delta(x - \lambda(n))g(x)\delta(x - \lambda(m)), \qquad (19)$$

where x is a dummy variable of integration to run over all admissible values of λ. Therefore, such terms with pair-wise different values of $\lambda(n)$ in Eq. (15), *i.e.*, whenever $n \neq m$, are, in accord with Eq. (10), identically zero under integration over λ. This annihilates two terms on the right side of

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[2] BELL's notation, e.g., $P(a,\lambda)$, makes no distinction between variables, here a, and conditioning parameters, here λ, customarily separated by a vertical bar rather than a comma. This oversight is the source of much confusion, and possibly even the subliminal cause of his 'error'. In this paper, Bell's notation is retained whenever referring directly to his formulas.

[3] BELL used a single symbol: λ, to denote what could be a complicated set of variables of possibly different types even. Thus, a "particular values for λ" means that each entity in the whole set must have a value.

Eq. (15), so that the final form of this Bell Inequality is then actually the trivial identity:

$$|P(a, b)| + |P(a', b')| \leq 2. \qquad (20)$$

Thus, our final conclusion is, that the proof of the ineluctability of the presence of nonlocality in QM, is invalid. In the context of what is actually a further point regarding the admissibility of additional, ostensibly still 'hidden' variable completions of QM, this conclusion undermines the popular impression that such a 'completion' necessarily cannot reinstate 'reality' and 'locality.'[4]

IV. BOHM'S VERSION OF EPR EXPERIMENTS

EPR proposed a gedanken experiment involving the disintegration of a mother particle into two daughters moving off in opposite directions.[7] They observed that quantum principles state that HAMILTONian conjugate variables suffer HEISENBERG uncertainty, and therefore, cannot be simultaneously determined exactly. But, in the situation envisioned by EPR, the experimenter could observe one daughter's position to arbitrarily high precision, while observing to arbitrarily high precision the other daughter's momentum; and, then, calling on symmetry, one could specify to arbitrary high precision all four quantities—in conflict with HEISENBERG's Principle. Hence, the paradox.

Complicating matters, actually doing the EPR experiment as proposed, is impractical. So, as is well known, BOHM proposed another venue, now called "qubit" space.[16] Originally he considered using particles with spin; but, this venue too was impractical, so experimenters chose spin's homeomorphic partner: light polarisation space. This change of venue, however, introduces two serious defects. One, the two states of polarisation, in spite of not commuting (for purely geometric reasons it turns out), are not HAMILTONian conjugate pairs; the structure involved in not intrinsic to QM, but just to classical electrodynamics. Two, the two polarisation states are non interacting; the structure leading to 'irreal' states as the sum of two mutually contradictory summands, is not relevant. Appropriate states for emissions of polarisation correlated daughter signals can be expressed without use of the 'irreal' format. In conclusion, even disregarding JAYNES' challenge to their general validity, various hypothetical inputs into the logic of the derivation of BELL inequalities have not been met by optical experiments widely credited with "proving BELL's theorem".[5]

V. SUMMARY AND FORECAST

Herein, history has been, so to say, "rewritten" so as to parse better the logical interrelationships among the features constraining development of an interpretation for QM. The concluding point here is that the assumption that quantum theory is "complete" compelled the introduction of the notions of 'irreality' and 'nonlocality'. That the assumption of 'completeness' is the source of conflicted logic is by far not here unique; POST has argued convincingly already for decades, on the basis of detailed analysis of numerous specific "quantum" phenomena, that this assumption introduces serious lacuna.[18]

Unfortunately, simply rejecting the completeness assumption alone does not render the matter clear. QM captures an undulatory feature even deeper than the structure involving the statistics of ensembles of atomic scale entities; this requires a physical nature for 'wave functions'; despite SCHRÖDINGER's failure to find it, they cannot simply be just progenitors of statistical densities. A full remedy must be found elsewhere.[6]

Independently and additionally, I argue in support of JAYNES, that BELL's analysis stemming from the EPR arguments in favour of the incompleteness of QM, contains an error.

Beyond the philosophical (or accompanying mathematical) implications of revamping the nowadays customary understanding of this issue in terms of "entanglement", there is a possible solidly practical consequence. It follows from the fact that the physical manifestations of entanglement targeted for exploitation, all depend in the end on the consequences of superposition resulting from the linearity of the hyperbolic differential equation underpinning QM, i.e., SCHRÖDINGER's equation. Since this hyperbolic structure is not an exclusive consequence of any quantum feature, certain applications now thought to require atomic scale realization, 'quantum logic gates' for example, may in fact not need micro devices to run what are thought to be 'quantum algorithms'. As manufacturing macroscopic devices is much easier, a computer realizing the parallelism of what turns out to be implicit in all FOURIER analysis, not just from 'quantum entanglement', might be eminently obtainable.[7]

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[4] KOCHEN-SPECKER type 'no-go' theorems without probabilities also can be shown to be defective as all their hypothetical inputs cannot be true simultaneously on physical grounds.[15]

[5] EPR experiments on particles have given largely ambiguous results for additional, independent reasons. See: [17] for a current review of EPR experiments, albeit without acknowledging JAYNES' arguments.

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[6] Not surprisingly, this writer prefers his own proposal for a resolution of the enigma of the origin of wavelike probabilistic features in QM.[19]

[7] This speculation assumes, of course, the physical cogency of 'quantum algorithms', a matter on which this writer holds no professional opinion as of yet. But there is room for optimism if these algorithms require only superposition and not irreality.

[8] The writer's English translations of references [1; 2], and preprints of [11; 12; 13; 19] can be downloaded from:
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